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Model Reference Adaptive Fuzzy Control System on an Aeroengine

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A method to design a model reference adaptive controller (MRAC) has been proposed for an aeroengine speed digital control system. The adaptive control device is designed using fuzzy control theory. A digital simulation of the entire control system has been executed and indicated that precision control of aeroengine speed is improved. Control algorithm complexity is reduced using this method. This new method is proposed as a tool to design MRACs for those systems that are high-order, nonlinear, or imprecisely modeled.

Introduction

SINCE the paper discussed in Ref. 1 was published in 1965 as a newly developing course of study, fuzzy set theory has been applied to areas of the national economy including earthquakes and industrial applications.[2,3]

Reference 4 is primarily concerned with research in the aeroengine control area. Both Refs. 4 and 5 indicate that systems using fuzzy control manifest good performance.

Although model reference adaptive controller (MRAC) is a valid control design method, there are many difficulties associated with its application. The method needs an approximate or precise mathematics model to be the object of study, and considerable control error must be produced in its application.[6] The algorithm of MRAC is complicated, therefore, its control process is inaccessible in real time, especially for the high-order, nonlinear system such as the aeroengine, therefore, we designed the MRACs using the fuzzy control theory to advance the performance of a control system.[7]

System Design

The control structure composition shown in Fig. 1 considered the virtue of the new fuzzy control method. Here the aeroengine is a two-rotor engine, the $\rho(t)$ is reference input and n_{lm} is the speed output of the reference model.

$$\rho(t) = \begin{cases} 4501.738 + 62.982x + 14.792x^2 - 6.2067x^3 \\ x = 8.3299t - 0.999, \quad t \le 0.24\ \text{s} \\ 4965.434 + 297.85x - 38.539x^2 + 55.710x^3 \\ x = 1.1364t - 1.273, \quad 0.24\ \text{s} < t \le 2.00\ \text{s} \\ \qquad \cdots \\ 11{,}157.43 + 0.9224x - 7.4555x^2 + 3.6817x^3 \\ x = 4.545t - 44.86, \quad 9.65\ \text{s} < t \le 10.09\ \text{s} \\ 11{,}157.06 + 5.2214x - 2.7609x^2 + 5.4997x^3 \\ x = 2.3256t - 24.47, \quad 10.09\ \text{s} < t \le 10.95\ \text{s} \\ 11{,}156.0, \quad 10.95\ \text{s} < t \end{cases}$$

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The transfer function of reference model $G_m(s)$:

$$G_m(s) = \frac{37.941919}{S^2 + 10.59664S + 37.941919}$$

the transfer function of the controlled object (engine) $G_p(s)$:

$$G_p(s) = \frac{\delta N_L}{\delta m_t} = \frac{K(\tau S + 1)D}{S^2 + CS + D}$$

The sectional linear small deviation engine model is adopted in this article. We divided the model into 34 sections from the idling state ($nl_p = 4462$ rpm) to the maximum speed state ($nl_p = 11{,}156$ rpm). The parameters of the engine model varied in different sections. The reference model is proposed for the response of the small deviation of the system and the reference input, which is a sectional continuous function made so that the reference model can be applied to satisfy the requirement of the big deviation of the engine.

In the system the input variables are the generalized error e and the first derivative of the generalized error $\dot{e}$. In the operation the fuzzy control device continuously samples e and $\dot{e}$ ($\dot{e}$ is obtained through calculation), which is further linearly transformed to the relevant E and $\dot{E}$ by the coefficients C_e and C_{de}. The control output U can be found as E and $\dot{E}$ and were carried to the fuzzy device. When U is multiplied by the coefficient C_u, the control output is obtained. The C_e, C_{de}, and C_u appeared over are determined on the basis of the requirement of the system to control quantity.

In order to raise the adaptive ability of the adaptive controller, we lead the self-rectifying part with the coefficient K to compensate the control performance of the system.

Algorithm of Adaptive Device Using Fuzzy Control Theory

The method of fuzzy control is different from other control methods, essentially because it is an intellect control method

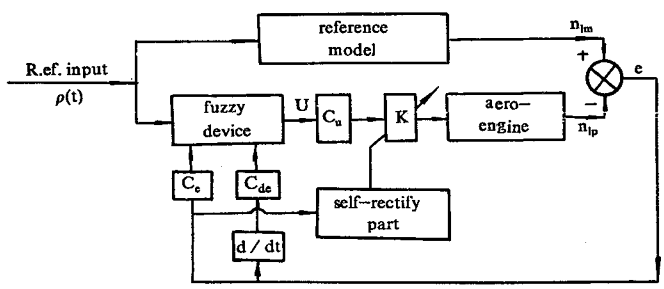

Fig. 1 Structure composition of MRAC system with a fuzzy device.

that reflects the actual demand of a human to the control system. Two kinds of methods are adopted to design the fuzzy control device, i.e., the phase–plane method and the membership grade method. The inference composition is on-line to the phase–plane method and the out-line calculation is adopted in the membership grade method.

Phase–Plane Method

The control rule of the method can be generalized by an analysis formula, i.e.,

$$U = \langle \alpha E + (1 - \alpha)\dot{E} \rangle$$

In the formula α is real and $0 < \alpha < 1$. The symbol $\langle\ \rangle$, such as $\langle \alpha \rangle$, indicated a minimum integer that has the same sign as α and the absolute value of which is bigger than or equal to the absolute value α. In some cases it is impossible to make the control performance to satisfy the system requirement if only one control parameter α is used, and so the control rule of multiple control parameter α is used in order to improve the control performance, i.e.,

$$U = \begin{cases} \langle \alpha_0 E + (1 - \alpha_0)\dot{E} \rangle, & \text{if } E = 0 \\ \langle \alpha_1 E + (1 - \alpha_1)\dot{E} \rangle, & \text{if } E = \pm 1 \\ \langle \alpha_2 E + (1 - \alpha_2)\dot{E} \rangle, & \text{if } E = \pm 2 \\ \vdots \\ \langle \alpha_6 E + (1 - \alpha_6)\dot{E} \rangle, & \text{if } E = \pm 6 \end{cases}$$

This kind of control rule has great mobility, it can satisfy the different demands of e and $\dot{e}$ in any case and make the control system obtain excellent performance. The simplex method was adopted to find the optimum parameters according to the individual and multiple parameter control rule, thus both a single and a group of parameters are obtained, in the process of which the performance index integral time times absolute error (ITAE) served as on objective function.

Membership Grade Method

Suppose that NB, NM, NS, NZ, Z, . . . , PB were used to express, respectively, the linguistic value negative big, negative middle, negative small, negative zero, zero, . . . , positive big; then the fuzzy control rule and the membership function of error are shown as Tables 1 and 2, respectively; the membership functions $\dot{E}$ and U are similar to Table 2.

Table 1 Fuzzy control rule

$\dot{E}$ \ E / $\underline{U}$	NB	NM	NS	NZ	PZ	PS	PM	PS
NB	PB	PB	PM	PM	PM	PS	Z	Z
NM	PB	PB	PM	PM	PM	PS	Z	Z
NS	PB	PB	PM	PS	PS	Z	NM	NM
Z	PB	PB	PS	Z	Z	NS	NB	NB
PS	PM	PM	Z	NS	NS	NM	NB	NB
PM	Z	Z	ZS	NM	NM	NM	NB	NB
PB	Z	Z	ZS	NM	NM	NM	NB	NB

The control rule can be expressed by the following fuzzy conditional sentences:

$$\begin{aligned} &\text{if } E_1 \text{ and } \dot{E}_1, \text{ then } \underline{U}_1 \\ &\text{if } E_2 \text{ and } \dot{E}_2, \text{ then } \underline{U}_2 \\ &\vdots \\ &\text{if } E_n \text{ and } \dot{E}_n, \text{ then } \underline{U}_n \end{aligned}$$

Corresponding to every fuzzy sentence, from the membership grade of E_i and $\dot{E}_i$, we can get

$$R_{E_i \times \dot{E}_i} = D_j^i = (D_1^i, D_2^i, \ldots, D_m^i)$$

In this formula D_j^i is the jth row vector, ($i = 1, 2, \ldots n$; $j = 1, 2, \ldots, m$). Making the D_j^i to a long column, we can get

$$D_j^i = \begin{bmatrix} D_1^i \\ D_2^i \\ D_3^i \\ \vdots \\ D_m^i \end{bmatrix}$$

The corresponding fuzzy relation matrix is

$$R_{E_i \times \dot{E}_i \times U_i} = D_j^i \cdot U_i$$

then

$$R_{E \times \dot{E} \times U} = \bigcup_{i=1}^{n} R_{E_i \times \dot{E}_i x U_i}$$

Utilizing the actual value and the fuzzy membership function we can get

$$D' = \begin{bmatrix} D_1' \\ D_2' \\ \vdots \\ D_m' \end{bmatrix}$$

Then using the fuzzy conversion the fuzzy control output $\underline{U}$ is obtained, i.e.,

$$\underline{U} = (D')^T \cdot R_{E \times \dot{E} \times \underline{U}}$$

In the concrete calculation, by investigating the relative membership grade table on the basis of the grade of E and $\dot{E}$, we can find all the linguistic variables corresponding to the control grade and find the membership function and rel-

Table 2 Membership function of error

$\dot{E}$ \ E / $\mu_i(E)$	−6	−5	−4	−3	−2	−1	−0	+0	+1	+2	+3	+4	+5	+6
NB	1.0	0.8	0.4	0.1	0	0	0	0	0	0	0	0	0	0
NM	0.2	0.7	1.0	0.7	0.2	0	0	0	0	0	0	0	0	0
NS	0	0	0.1	0.5	1.0	0.8	0.3	0	0	0	0	0	0	0
NZ	0	0	0	0	0.1	0.6	1.0	0	0	0	0	0	0	0
PZ	0	0	0	0	0	0	0	1.0	0.6	0.1	0	0	0	0
PS	0	0	0	0	0	0	0	0.3	0.8	1.0	0.5	0.1	0	0
PM	0	0	0	0	0	0	0	0	0	0.2	0.7	1.0	0.7	0.2
PM	0	0	0	0	0	0	0	0	0	0	0.1	0.4	0.8	1.0

Table 3 Actual control output by center area method

$\dot{E}$ \ E / U	−6	−5	−4	−3	−2	−1	−0	+0	+1	+2	+3	+4	+5	+6
−6	6.0	5.5	5.5	4.5	3.5	3.5	3.5	3.0	2.0	1.0	0.5	−0.5	−0.5	−0.5
−5	6.0	5.5	5.5	4.5	3.5	3.5	3.5	3.0	2.0	1.0	0.5	−0.5	−0.5	−0.5
−4	6.0	5.5	5.5	4.5	3.5	3.0	3.5	3.0	2.0	0.5	−0.5	−1.0	−1.0	−1.0
−3	6.0	5.5	5.5	4.5	3.5	2.5	2.5	2.0	1.5	0	−2.0	−3.0	−3.5	−3.5
−2	6.0	5.5	5.5	4.5	3.5	2.5	1.5	1.0	0.5	−0.5	3.0	−4.0	−4.0	−4.0
−1	6.0	5.5	5.5	3.5	3.0	2.0	1.0	0.5	−0.5	−1.5	−4.0	−5.0	−5.0	−5.0
0	6.0	5.5	5.5	3.5	1.0	0.5	0	−1.0	−1.5	−2.0	−4.5	−6.5	−6.5	−7.0
+1	4.0	4.0	4.0	3.0	0.5	−0.5	−1.5	−2.0	−3.0	−4.0	−4.5	−6.5	−6.5	−7.0
+2	3.0	3.0	3.0	2.0	−0.5	−1.5	−2.0	−2.5	−3.5	−4.5	−5.5	−6.5	−6.5	−7.0
+3	2.5	2.5	2.0	1.0	−1.0	−2.5	−3.0	−3.5	−3.5	−4.5	−5.5	−6.5	−6.5	−7.0
+4	0	0	0	−0.5	−1.5	−3.0	−4.0	−4.5	−4.0	−4.5	−5.5	−6.5	−6.5	−7.0
+5	−0.5	−0.5	−0.5	−1.5	−2.0	−3.0	−4.0	−4.5	−4.5	−4.5	−5.5	−6.5	−6.5	−7.0
+6	−0.5	−0.5	−0.5	−1.5	−2.0	−3.0	−4.0	−4.5	−4.5	−5.5	−6.5	−6.5	−6.5	−7.0

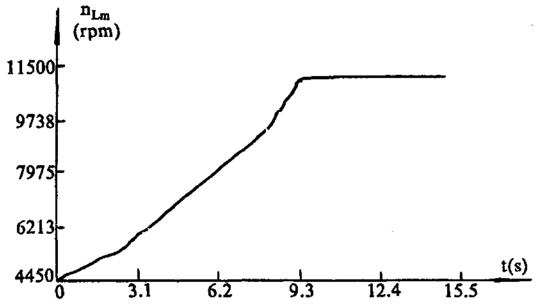

Fig. 2 Reference output of system applying a big deviation.

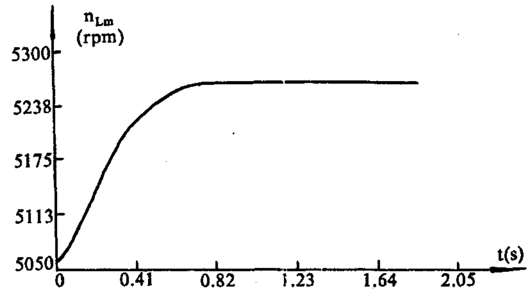

Fig. 3 Reference output of system applying a small deviation.

atively conditional control sentence. On the basis of the previously mentioned method we will get the fuzzy control output. Through the fuzzy decision the fuzzy control output can be converted to actual control output. Table 3 is the actual control output on the basis of the fuzzy decision of the "center area method."

Self-Rectification

Due to the uncertainty of the engine model and the speciality of model parameters that varied continuously, it is impossible to satisfy the design requirement of the control system at any time by the fuzzy controller. For this reason the gain self-adjust rule is led into the adaptive control system of the engine speed to rectify the parameters in this article. It made the controller more powerful and adaptive to the change of the engine-specific property, so that the control effect tended to be optimum.

The self-rectifying part was acted on by the input of the engine. If the self-rectifying coefficient, K is big, the steady error of the system is reduced, improving the control precision and speeding up the response, but just because of the excessively big k, the response will be overshot. On the contrary, if the K is excessively small, the stability can be improved, also just because of being excessively small, the transient process will be prolonged. In order to get a better rectifying coefficient k, let

$$J_k = \frac{1}{me_{\text{inf}}}\left(\sum_{i=1}^{m} |e_{k-i}|\right)$$

In this formula, k is the current moment, e_{k-i} is the control error at $k - i$ moment, e_{inj} is the inferior precision to the control system, m is the delay time, and $m < k$.

From the previously mentioned formula, when the average of error satisfied the precision in finite time, there must be $J_k \leqslant 1.0$, the system needs a small control output, correspondingly. Otherwise, $J_k \geqslant 1.0$, and the system needs a bigger control output. For this reason we chose the J_k as the following formula:

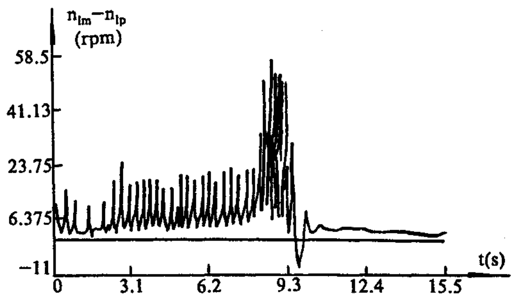

Fig. 4 Response ($n_{l_m} - n_{l_p}$) of the system applying a big deviation in simulation (using phase–plane method).

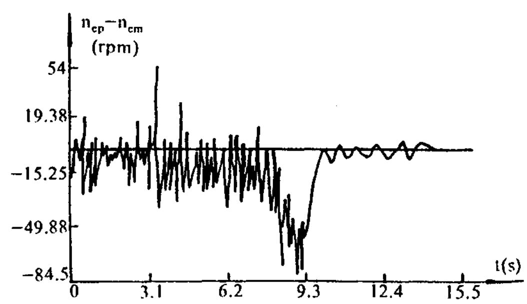

Fig. 5 Response ($n_{l_p} - n_{l_m}$) of the system applying a big deviation in simulation (using membership grade method).

$$K = \begin{cases} 0 & \text{when } j_k \leqslant 0.5 \\ J_k, & \text{when } 0.5 < J_k \leqslant 2.0 \\ 2.0J_k, & \text{when } 2.0 < J_k \leqslant 5.0 \\ 5.0J_k, & \text{when } 5.0 < J_k \end{cases}$$

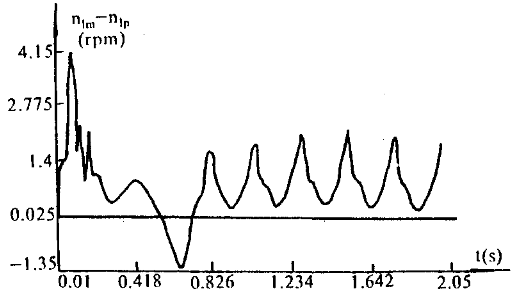

Fig. 6 Response (n_{lm} − n_{lp}) of a system applying a small deviation in simulation (using phase–plane method).

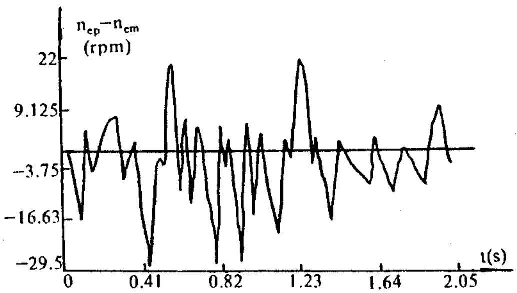

Fig. 7 Response (n_{lp} − n_{lm}) of a system applying a small deviation in simulation (using membership grade method).

In general, m is 1–5, and the excessively big m is not suitable. So, the self-rectifying part can force the controller to increase or decrease the control output in a big range.

Digital Simulation and Results Analysis

In this article the digital simulation of the entire control system is made with C language. The reference output of the system that has applied a big deviation and small deviation is shown as Figs. 2 and 3, respectively. The simulation results of the system that has applied a big deviation have adopted the phase–plane method and membership grade method (using the center of area method as the fuzzy decision). The phase–plane method used multiple control parameters, i.e., $\alpha_0, \alpha_1, \ldots, \alpha_6$ by the simplex and the membership grade methods shown in Figs. 4 and 5, respectively. In the same case, the results of a small deviation were shown in Figs. 6 and 7.

Overall, considering the control effects of the big and small deviations, the simulation result of the phase–plane method is superior to that of the membership grade method.

Conclusions

Using the fuzzy set theory to design adaptive control devices, the precise mathematics model establishment of the controlled object are not necessary. Only by depending on the fuzzy information under the definite control target using the fuzzy condition sentence to obtain control rule, can the good control effect be gotten. Compare this to a normal adaptive algorithm; using this adaptive algorithm and avoiding the complicated inferring process, sum up the simple calculation or checking table, it takes little time to calculate in a sampling period, so that it is possible to realize the real time control. The author had carried out the real-time simulation with an 8098 single-chip computer that imitated the engine and the personal computer that imitated the controller. It proposed a valid method to design an adaptive control device for the MRAC.

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